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Technical Notes

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Observability of Self-Sensing System Using Extended Kalman Filter

Kanjuro Makihara,* Junjiro Onoda,† and Takuya Yabu‡
Japan Aerospace Exploration Agency, Kanagawa 229-8510, Japan

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Nomenclature

$\boldsymbol{B}_p$	=	input matrix ($l \times m$)
$\boldsymbol{C}_p$	=	diagonal piezoelectric capacitance matrix ($m \times m$)
$\boldsymbol{I}_n$	=	unit matrix ($n \times n$)
$\boldsymbol{K}$	=	constant-charge stiffness matrix ($l \times l$)
l	=	number of DOF of structure
$\boldsymbol{M}$	=	mass matrix ($l \times l$)
m	=	number of piezoelectric actuators
$\boldsymbol{O}_{k\times n}$	=	zero matrix ($k \times n$)
$\boldsymbol{Q}$	=	charge vector ($m \times 1$)
$\boldsymbol{V}$	=	voltage vector ($m \times 1$)
$\boldsymbol{w}$	=	external force vector ($l \times 1$)
$\boldsymbol{x}$	=	displacement vector ($l \times 1$)
$\boldsymbol{z}$	=	state vector in Eq. (7) ($2l \times 1$)
$\boldsymbol{z}_e$	=	extended state vector in Eq. (10) [$(2l + m) \times 1$]
Γ_e	=	observer gain matrix [$(2l + m) \times m$]
$\boldsymbol{\Xi}$	=	diagonal modal damping matrix ($l \times l$)
$\boldsymbol{\xi}$	=	modal displacement vector ($l \times 1$)
$\boldsymbol{\Phi}$	=	eigen matrix ($l \times l$)
$\boldsymbol{\Omega}$	=	diagonal constant-charge modal stiffness matrix ($l \times l$)

I. Introduction

A novel self-sensing method using piezoelectric actuators for semiactive vibration suppression was proposed [1]. By using extended system equations, this self-sensing method can be implemented with the Kalman filter [2]. Experiments and numerical simulations demonstrated the self-sensing method has significantly good suppression performance. The self-sensing method separated electrical status into two cases, $\dot{\boldsymbol{Q}} = 0$ and $\dot{\boldsymbol{Q}} \neq 0$, and characterized each of these. However, because the former case has the extended matrix $\boldsymbol{A}_e$ that has rows filled with zeros, an issue concerning nonobservability may arise. The previous paper [1] has not discussed on this essential issue, and this note thus discusses the observability of the self-sensing method.

II. Equations for Structure with Piezoelectric Actuators

A brief description of formulae for the self-sensing method is presented to enable a better understanding. The previous paper [1] presents the multidegree-of-freedom (MDOF) structure having multiple piezoelectric actuators. The equation of motion is

$$\boldsymbol{M}\ddot{\boldsymbol{x}} + \boldsymbol{K}\boldsymbol{x} - \boldsymbol{B}_p\boldsymbol{Q} - \boldsymbol{w} = 0 \tag{1}$$

The modal equation with the modal damping matrix can be expressed as

$$\ddot{\boldsymbol{\xi}} + \boldsymbol{\Xi}\dot{\boldsymbol{\xi}} + \boldsymbol{\Omega}\boldsymbol{\xi} - \boldsymbol{\Phi}^T\boldsymbol{B}_p\boldsymbol{Q} - \boldsymbol{\Phi}^T\boldsymbol{w} = 0 \tag{2}$$

and the voltage equation is written as

$$\boldsymbol{V} = -\boldsymbol{B}_p^T\boldsymbol{\Phi}\boldsymbol{\xi} + \boldsymbol{C}_p^{-1}\boldsymbol{Q} \tag{3}$$

where

$$\boldsymbol{\Phi} \equiv [\boldsymbol{\phi}_1, \boldsymbol{\phi}_2, \cdots, \boldsymbol{\phi}_l], \qquad \boldsymbol{\Omega} \equiv \mathrm{diag}[\omega_k^2], \qquad \boldsymbol{\Xi} \equiv \mathrm{diag}[2\zeta\omega_k] \quad (k = 1, 2, \cdots, l) \tag{4}$$

Equations (2) and (3) can be transformed into

$$\dot{\boldsymbol{z}} = \boldsymbol{A}\boldsymbol{z} + \boldsymbol{B}\boldsymbol{Q} + \boldsymbol{E}\boldsymbol{w}, \qquad \boldsymbol{V} = \boldsymbol{C}\boldsymbol{z} + \boldsymbol{D}\boldsymbol{Q} \tag{5}$$

where

$$\boldsymbol{A} \equiv \begin{bmatrix} \boldsymbol{0}_{l\times l} & \boldsymbol{I}_l \\ -\boldsymbol{\Omega} & -\boldsymbol{\Xi} \end{bmatrix}, \qquad \boldsymbol{B} \equiv \begin{bmatrix} \boldsymbol{0}_{l\times m} \\ \boldsymbol{\Phi}^T\boldsymbol{B}_p \end{bmatrix}, \qquad \boldsymbol{C} \equiv [-\boldsymbol{B}_p^T\boldsymbol{\Phi} \quad \boldsymbol{0}_{m\times l}] \tag{6}$$

$$\boldsymbol{D} \equiv \boldsymbol{C}_p^{-1}, \qquad \boldsymbol{E} \equiv \begin{bmatrix} \boldsymbol{0}_{l\times l} \\ \boldsymbol{\Phi}^T \end{bmatrix}, \qquad \boldsymbol{z} \equiv [\boldsymbol{\xi}^T, \dot{\boldsymbol{\xi}}^T]^T \tag{7}$$

In the case that $\dot{\boldsymbol{Q}} = \boldsymbol{0}$, Eq. (5) can be transformed into

$$\dot{\boldsymbol{z}}_e = \boldsymbol{A}_e\boldsymbol{z}_e + \boldsymbol{E}_e\boldsymbol{w}, \qquad \boldsymbol{V} = \boldsymbol{C}_e\boldsymbol{z}_e \tag{8}$$

where

$$\boldsymbol{A}_e \equiv \begin{bmatrix} \boldsymbol{A} & \boldsymbol{B} \\ \boldsymbol{0}_{m\times 2l} & \boldsymbol{0}_{m\times m} \end{bmatrix}, \qquad \boldsymbol{E}_e \equiv \begin{bmatrix} \boldsymbol{E} \\ \boldsymbol{0}_{m\times l} \end{bmatrix} \tag{9}$$

$$\boldsymbol{C}_e \equiv [\boldsymbol{C} \quad \boldsymbol{D}], \qquad \boldsymbol{z}_e \equiv [\boldsymbol{z}^T, \boldsymbol{Q}^T]^T \tag{10}$$

The Kalman filter [2] for estimating $\boldsymbol{z}_e$ is

$$\dot{\hat{\boldsymbol{z}}}_e = \boldsymbol{A}_e\hat{\boldsymbol{z}}_e + \Gamma_e(\boldsymbol{V} - \boldsymbol{C}_e\hat{\boldsymbol{z}}_e) \tag{11}$$

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*Aerospace Project Research Associate, Department of Space Structure and Materials, Institute of Space and Astronautical Science (ISAS), 3-1-1 Yoshinodai, Sagamihara; kanjuro@svs.eng.isas.jaxa.jp. Member AIAA.

†Professor, Department of Space Structure and Materials, Institute of Space and Astronautical Science (ISAS), 3-1-1 Yoshinodai, Sagamihara. Associate Fellow AIAA.

‡Research Fellow, Department of Space Structure and Materials, Institute of Space and Astronautical Science (ISAS), 3-1-1 Yoshinodai, Sagamihara.

The matrix dimensions are as follows: $\boldsymbol{A}$: $2l \times 2l$, $\boldsymbol{B}$: $2l \times m$, $\boldsymbol{C}$: $m \times 2l$, $\boldsymbol{D}$: $m \times m$, $\boldsymbol{A}_e$: $(2l+m) \times (2l+m)$, and $\boldsymbol{C}_e$: $m \times (2l+m)$. $\boldsymbol{\Omega}$, $\boldsymbol{\Xi}$, and $\boldsymbol{D}$ are diagonal and positive-definitive matrices.

III. Observability of Self-Sensing System

This note selects the PBH (Popov, Belevitch, and Hautus) rank test [3] to examine the observability of the self-sensing system.

A. PBH Rank Test

Theorem: The system in Eq. (8) is observable, if the following $(2l+2m) \times (2l+m)$ complex matrix has a rank of $(2l+m)$ for every eigenvalue λ of $\boldsymbol{A}_e$.

$$\boldsymbol{H} \equiv \begin{bmatrix} \lambda \boldsymbol{I}_{2l+m} - \boldsymbol{A}_e \\ \boldsymbol{C}_e \end{bmatrix} \tag{12}$$

B. Rank Calculation

It is easily shown that $\boldsymbol{A}_e$ in Eq. (9) has $(2l+m)$ eigenvalues, i.e., $2l$ pieces of λ_1 and m pieces of λ_2 where

$$\lambda_1 \equiv \omega_k(-\zeta \pm i\sqrt{1-\zeta^2}), \qquad \lambda_2 \equiv 0 \qquad (k = 1, 2, \cdots, l) \tag{13}$$

Firstly, substituting $\lambda_1 (\neq 0)$ into $\boldsymbol{H}$ in Eq. (12), we obtain

$$\boldsymbol{H}_1 \equiv \begin{bmatrix} \lambda_1 \boldsymbol{I}_l & -\boldsymbol{I}_l & \boldsymbol{0}_{l\times m} \\ \boldsymbol{\Omega} & \lambda_1 \boldsymbol{I}_l + \boldsymbol{\Xi} & -\boldsymbol{\Phi}^T \boldsymbol{B}_p \\ \boldsymbol{0}_{m\times l} & \boldsymbol{0}_{m\times l} & \lambda_1 \boldsymbol{I}_m \\ -(\boldsymbol{\Phi}^T \boldsymbol{B}_p)^T & \boldsymbol{0}_{m\times l} & \boldsymbol{D} \end{bmatrix} \tag{14}$$

To determine the rank of $\boldsymbol{H}_1$, we transform it in the sense of the elementary transformation. Multiplying by elementary transformation matrices, $\boldsymbol{H}_1$ becomes

$$\boldsymbol{S}_5 \boldsymbol{S}_4 \boldsymbol{S}_3 \boldsymbol{S}_2 \boldsymbol{S}_1 \boldsymbol{H}_1 = \begin{bmatrix} \boldsymbol{I}_l & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{I}_l & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{I}_m \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0}_{m\times m} \end{bmatrix} \tag{15}$$

as long as $\lambda_1 \neq 0$. It is thus that

$$\text{rank}\,(\boldsymbol{H}_1) = 2l + m \tag{16}$$

In brief, $\boldsymbol{H}_1$ has a rank of $(2l+m)$ unconditionally.

Secondly, substituting λ_2 into $\boldsymbol{H}$ in Eq. (12), we obtain

$$\boldsymbol{H}_2 \equiv \begin{bmatrix} \boldsymbol{0}_{l\times l} & -\boldsymbol{I}_l & \boldsymbol{0}_{l\times m} \\ \boldsymbol{\Omega} & \boldsymbol{\Xi} & -\boldsymbol{\Phi}^T \boldsymbol{B}_p \\ \boldsymbol{0}_{m\times l} & \boldsymbol{0}_{m\times l} & \boldsymbol{0}_{m\times m} \\ -(\boldsymbol{\Phi}^T \boldsymbol{B}_p)^T & \boldsymbol{0}_{m\times l} & \boldsymbol{D} \end{bmatrix} \tag{17}$$

Multiplying by elementary transformation matrices, $\boldsymbol{H}_2$ becomes

$$\boldsymbol{P}_4 \boldsymbol{P}_3 \boldsymbol{P}_2 \boldsymbol{P}_1 \boldsymbol{H}_2 \boldsymbol{Q}_1 = \begin{bmatrix} \boldsymbol{I}_l & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{I}_l & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{J}_m \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0}_{m\times m} \end{bmatrix} \tag{18}$$

The rank of $\boldsymbol{H}_2$ depends on the rank of $\boldsymbol{J}_m$, where

$$\boldsymbol{J}_m \equiv \boldsymbol{I}_m - \boldsymbol{D}^{-1}(\boldsymbol{\Phi}^T \boldsymbol{B}_p)^T \boldsymbol{\Omega}^{-1}(\boldsymbol{\Phi}^T \boldsymbol{B}_p) = \boldsymbol{I}_m - \boldsymbol{C}_p \boldsymbol{B}_p^T \boldsymbol{K}^{-1} \boldsymbol{B}_p \tag{19}$$

On the condition that

$$\text{rank}\,(\boldsymbol{J}_m) = m \tag{20}$$

it is valid that

$$\text{rank}\,(\boldsymbol{H}_2) = 2l + m \tag{21}$$

In conclusion, Eqs. (16) and (21) prove that the extended system in Eq. (8) is observable as long as $\text{rank}(\boldsymbol{J}_m) = m$.

$\boldsymbol{J}_m$ is composed of the piezoelectric capacitance matrix $\boldsymbol{C}_p$, the input matrix $\boldsymbol{B}_p$, and the constant-charge stiffness matrix $\boldsymbol{K}$. These three matrices contain independent parameters for an MDOF structure having piezoelectric actuators. Therefore, it is just coincidence when the rank of $\boldsymbol{J}_m$ degrades from m. This note presents a discriminant matrix, $\boldsymbol{J}_m$, which can be used to examine the observability of the self-sensing system.

C. Simple Case Analysis

For a better understanding of the discriminant matrix, let us consider a simple system having a single DOF and a single piezoelectric actuator, i.e., $l = m = 1$. Three matrices, $\boldsymbol{C}_p$, $\boldsymbol{B}_p$, and $\boldsymbol{K}$, turn to scalars, C_p, b_p, and k, respectively. We obtain

$$\boldsymbol{J}_1 = (k - b_p^2 C_p)/k \tag{22}$$

The observability depends on whether $\boldsymbol{J}_1$ is zero or not. Here, $(k - b_p^2 C_p)$ is the constant-voltage stiffness, whereas k is the constant-charge stiffness, as discussed in [4]. It is mathematically possible that the constant-voltage stiffness is zero, but it is quite unlikely to occur in actual systems. Therefore, this simple system's observability is thus ensured in practice.

IV. Conclusions

This note discussed the observability of the self-sensing system that the authors had proposed. Through mathematical analysis based on the PBH rank test, the discriminant matrix was derived that determines the observability of the self-sensing system. The analytical investigation presented here gave the self-sensing method a guarantee for actual applications.

Appendix: Transformation Matrices

Some transformation matrices are defined as follows:

$$\boldsymbol{S}_1 \equiv \begin{bmatrix} (1/\lambda_1)\boldsymbol{I}_l & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{I}_l & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & (1/\lambda_1)\boldsymbol{I}_m & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{I}_m \end{bmatrix},$$

$$\boldsymbol{S}_2 \equiv \begin{bmatrix} \boldsymbol{I}_l & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ -\boldsymbol{\Omega} & \boldsymbol{I}_l & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{I}_m & \boldsymbol{0} \\ (\boldsymbol{\Phi}^T \boldsymbol{B}_p)^T & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{I}_m \end{bmatrix},$$

$$\boldsymbol{S}_3 \equiv \begin{bmatrix} \boldsymbol{I}_l & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{I}_l & \boldsymbol{\Phi}^T \boldsymbol{B}_p & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{I}_m & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & -\boldsymbol{D} & \boldsymbol{I}_m \end{bmatrix}, \qquad \boldsymbol{S}_4 \equiv \begin{bmatrix} \boldsymbol{I}_l & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \Gamma^{-1} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{I}_m & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{I}_m \end{bmatrix},$$

$$\boldsymbol{S}_5 \equiv \begin{bmatrix} \boldsymbol{I}_l & (1/\lambda_1)\boldsymbol{I}_l & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{I}_l & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{I}_m & \boldsymbol{0} \\ \boldsymbol{0} & (1/\lambda_1)(\boldsymbol{\Phi}^T \boldsymbol{B}_p)^T & \boldsymbol{0} & \boldsymbol{I}_m \end{bmatrix},$$

$$\boldsymbol{P}_1 \equiv \begin{bmatrix} \boldsymbol{I}_l & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{\Xi} & \boldsymbol{I}_l & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{I}_m & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{I}_m \end{bmatrix}, \qquad \boldsymbol{P}_2 \equiv \begin{bmatrix} \boldsymbol{I}_l & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{\Omega}^{-1} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{I}_m & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{D}^{-1} \end{bmatrix},$$

$$\boldsymbol{P}_3 \equiv \begin{bmatrix} \boldsymbol{I}_l & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{I}_l & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{I}_m & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{D}^{-1}(\boldsymbol{\Phi}^T \boldsymbol{B}_p)^T & \boldsymbol{0} & \boldsymbol{I}_m \end{bmatrix},$$

$$\boldsymbol{P}_4 \equiv \begin{bmatrix} \boldsymbol{0} & \boldsymbol{I}_l & \boldsymbol{0} & \boldsymbol{0} \\ -\boldsymbol{I}_l & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{I}_m \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{I}_m & \boldsymbol{0} \end{bmatrix}, \qquad \boldsymbol{Q}_1 \equiv \begin{bmatrix} \boldsymbol{I}_l & \boldsymbol{0} & \boldsymbol{\Omega}^{-1}\boldsymbol{\Phi}^T \boldsymbol{B}_p \\ \boldsymbol{0} & \boldsymbol{I}_l & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{I}_m \end{bmatrix},$$

$$\Gamma \equiv (1/\lambda_1)\boldsymbol{\Omega} + \boldsymbol{\Xi} + \lambda_1 \boldsymbol{I}_l$$

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M. Ahmadian
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